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Original Article

GENERATING DSS GRAPH BY EDGE SUBDIVISION AND EDGE CONTRACTION

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ABSTRACT

A graph G is said to be domination subdivision stable (DSS) if γ (G_{sd} uv) = γ (G), for all u, v $\in$ V (G), u adjacent to v. In this paper we have provided two methods of obtaining a DSS graph from a non DSS graph.

Keywords: domination, subdivision, contraction.

INTRODUCTION

A set of vertices D in a graph G = (V, E) is a dominating set if every vertex of V – D is adjacent to some vertex of D. The cardinality of the smallest dominating set of G is called the domination number of G and it is denoted by γ (G). A vertex in V – D is k – dominated if it is dominated by at least k – vertices in D. For properties related to graph theory we refer to [1]

The open neighborhood of vertex v $\in$ V (G) is defined by N (v) = { u $\in$ V (G) | (u v) $\in$ E (G) } while its closed neighborhood is the set N [v] = N (v) $\cup$ { v }. The private neighborhood of v $\in$ D is defined by pn [v, D] = N (v) – N (D – { v }). We indicate that u is adjacent to v by writing u $\perp$ v. For properties related to domination we refer to [2].

MATERIALS AND METHODS

An elementary edge contraction of a graph G is obtained by removal of u and v and the addition of a new point w adjacent to those points which u or v was adjacent. G**.** uv is the graph obtained by contracting uv. In [3], Tamara Burton et al. defined a graph to be domination dot critical (DDC) if γ (G**.**uv) < γ (G), $\forall$ u, v $\in$ V (G). They have proved the following result.

The subdivision of some edge e with endpoints { u, v } yields a graph containing one new vertex w, and with an edge set replacing e by two new edges, { u, w } and { w, v }. We shall denote the graph obtained by subdividing any edge uv of a graph G, by G_{sd} uv. Let w be a vertex introduced by subdividing uv. We shall denote this by G_{sd} uv = w.

In [4], M. Yamuna et al have introduced the concept of domination subdivision stable graphs. A graph G is said to be domination subdivision stable (DSS) if the γ - value of G does not change by subdividing any edge of G.

Example

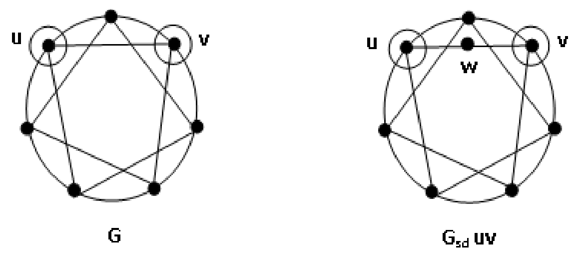

Fig. 1: γ **(G) =** γ **(** G_{sd} **uv) = 2. This is true for all e = ab** $\in$ **E (G). Here G is a DSS graph.**

In [4], they have proved the following result

R_1. A graph G is DSS if and only if for every u, v $\in$ V (G), either there is a γ - set containing u and v or, there is a γ - set D such that

1. pn[u, D] = { v } or
2. v is 2 – dominated.

R_2. Every DDC graph is DSS.

RESULTS AND DISCUSSIONS

Theorem 1

Let G be a graph which is not DSS. There is a subdivision graph of G which is DSS.

Proof

Since G is not DSS, there is at least one u, v $\in$ V (G), u $\perp$ v such that γ (G_{sd} uv) $\neq$ γ (G). By [R_1], either both u, v $\notin$ D or pn[u, D] $\geq$ 2, v $\in$ pn[u, D], where D is any γ – set for G. Let E = { e_s / e_s = (u_s, v_s), $u_s \perp v_s$, γ (G_{sd} u_sv_s) $\neq$ γ (G) }.

Let X_i = N (v_i) – { u_i }, Y_i = N (X_i), Z_i = N (u_i) – { v_i }. For any $e_m \in$ E (G), let γ (G_{sd} u_mv_m) = w_m. D_m = D $\cup$ { w_m } is a γ – set for G_{sd} u_mv_m. Let G_m = G_{sd} u_mv_m.

Case 1

$e_m \in$ E such that $u_m \in$ D, $v_m \notin$ D, m = 1, 2, …, k.

For all $x_m \in X_m$, γ ($G_{m\ sd}$ v_mx_m) = γ(G_m), since D_m – { w_m } $\cup$ { v_m } is a γ – set for G_m such that every $x_m \in X_m$, $x_m \in$ D is 2 – dominated.

For all $y_m \in Y_m$, such that $y_m \in$ D, γ ($G_{m\ sd}$ x_my_m) = γ(G_m), since D_m – { w_m } $\cup$ { x_m } is a γ – set for G_m for every $x_m \in X_m$. Also γ ($G_{m\ sd}$ u_mw_m) = γ ($G_{m\ sd}$ w_mv_m) = γ (G_m), since pn[w_m, D] = v_m.

Case 2

$e_m \in$ E such that u_m, $v_m \notin$ D, m = 1, 2, …, p. Note that k + p = | E |.

For all $x_m \in X_m$, γ ($G_{m\ sd}$ v_mx_m) = γ(G_m), since D_m – { w_m } $\cup$ { v_m } is a γ – set for G_m such that every $x_m \in X_m$, $x_m \notin$ D is 2 – dominated.

Similarly for all $z_m \in Z_m$, γ ($G_{m\ sd}$ u_mz_m) = γ(G_m), since D_m – { w_m } $\cup$ { u_m } is a γ – set for G_m such that every $z_m \in Z_m$, zm $\notin$ D is 2 – dominated. Also since w_m is selfish, γ ($G_{m\ sd}$ u_mw_m) = γ ($G_{m\ sd}$ w_mv_m) = γ (G_m).

From case 1, for all $e_m \in$ E (G) in graph G_m, for all $x_m \in X_m$, $y_m \in Y_m$ such that γ (G_{sd} v_mx_m) $\neq$ γ (G), γ (G_{sd} x_my_m) $\neq$ γ (G) become DSS in G_m.

Similarly from case 2, for all $e_m \in E (G)$ in a graph G_m, for all $x_m \in X_m$, $z_m \in Z_m$ such that $\gamma (G_{sd}\, v_m x_m) \neq \gamma (G)$ and $\gamma (G_{sd}\, u_m z_m) \neq \gamma (G)$ become DSS in G_m.

Also the new edges introduced are DSS and the edges which were subdivision stable in G are DSS in G_m also. Let $E_m = \{ e_m / e_m = (u_m\, v_m), u_m \perp v_m, \gamma (G_{sd}\, u_m v_m) \neq \gamma (G_m) \}$, clearly $| E_m | < | E |$. If G_m is DSS we terminate here else starting from E_m we repeat the same procedure to obtain a new graph G_{m+1}. If G_{m+1} is DSS we terminate here else we continue to generate a sequence of graphs $G_m, G_{m+1}, \ldots$ such that

1. $| E_{m+1} | < | E_m |$.
2. $\gamma (G_{m+1}) = \gamma (G_m) + 1$.

until we obtain a graph that is DSS.

Theorem 2

Let G be a graph which is not DSS. There is a graph H of G which is DSS, if H can be obtain from G by a sequence of elementary contraction.

Proof

Since G is not DSS there is at least one $u, v \in V (G)$, $u \perp v$ such that $\gamma (G_{sd}\, uv) \neq \gamma (G)$. By [R_1], either both $u, v \notin D$ or $pn[u, D] \geq 2$, $v \in pn[u, D]$, where D is any γ – set for G. Let $E = \{ es\}$ and $| E | = k + p$. Let $X_i = N (v_i) - \{ u_i \}$, $Y_i = N (X_i)$, $Z_i = N (u_i) - \{ v_i \}$. For any $e_m \in E (G)$, let $\gamma (G_{\bullet}\, u_m v_m) = \gamma (G)$, by [R_2]. D is a γ – set for $G_{\bullet}\, u_m v_m$ also. Let $G_m = G_{\bullet}\, u_m v_m$.

Case 1

$e_m \in E$ such that $u_m \in D$, $v_m \notin D$, where m = 1, 2, …, k.

For all $x_m \in X_m$, $\gamma (G_{m\ sd} (u_m v_m) x_m) = \gamma (G_m)$, since x_m is 2 – dominated in G_m.

For all $y_m \in Y_m$ such that $y_m \in D_m$, $\gamma (G_{m\ sd}\, x_m y_m) = \gamma (G_m)$, since x_m is 2 – dominated in G_m.

Case 2

$e_m \in E$ such that $u_m, v_m \notin D$, where m = 1, 2, …, p. Note that $k + p = | E |$.

For all $x_m \in X_m$, such that $x_m \in D_m$. $\gamma (G_{m\ sd} (u_m v_m) x_m) = \gamma (G_m)$, since $u_m v_m$ is 2 – dominated in G_m.

Similarly for all $z_m \in Z_m$ such that $z_m \in D_m$, $\gamma (G_{m\ sd} (u_m v_m) z_m) = \gamma (G_m)$, since $u_m v_m$ is 2 – dominated in G_m. The edges which are DSS in G are DSS in G_m also. Let $E_m = \{ e_m / e_m = (u_m\, v_m), u_m \perp v_m, \gamma (G_{m\ sd}\, u_m v_m) \neq \gamma (G) \}$, clearly $| E_m | < | E |$. If G_m is DSS, we terminate here else starting from E_m we repeat the same procedure to obtain a new graph G_{m+1}. If G_{m+1} is DSS we terminate here else we continue to generate a sequence of graphs $G_m, G_{m+1}, G_{m+2}, \ldots, G_q$ such that

1. $| E_{m+1} | < | E_m |$.
2. $\gamma (G_{m+1}) = \gamma (G_m)$.

until we obtain a graph G_{q+1} such that $\gamma (G_{q+1\bullet}\, uv) < \gamma (G_{q+1})$, for all $u, v \in V (G_{q+1})$, $u \perp v$, that is till we obtain a dot critical graph G_q. We know that every dot critical graph is DSS. This implies G_q is DSS.

CONCLUSION

The paper provides two methods of obtaining DSS graph from a non DSS graph by applying graph operations. By applying other graph operations like complement, dual, cross product we can generate DSS graphs from non DSS graphs.

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